

Great Walls of Water: The Pandemonium and Phenomena of Rogue Waves

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Abstract

This work employs the Nonlinear Schrödinger Equation (NLSE) to study instability in nonlinear systems—key factors in rogue wave generation. In addition, Gerstner’s equation is used to model the complex dynamics of these phenomena. The nonlinear effects associated with wave dynamics have been simulated using MATLAB, and Gerstner’s wave theory has been applied to model rogue wave behavior. The results from our investigations contribute to a better understanding of rogue waves and improved modeling of whether they are predictable, thereby enhancing maritime safety.

Keywords: Fluid Dynamics, NLSE, Gravity Waves, Exact Solutions

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1. Introduction

Rogue waves have been a major subject of both maritime legends and scientific inquiry for thousands of years. Spontaneously formed and extremely large, these waves can have heights of more than two and a half times the average height of the surrounding sea state, threatening ships very suddenly [1]. Rogue waves may appear abruptly, posing an imminent danger to vessels and leaving them with little to no recourse. Beyond their spontaneous creation and immense power, the mystery of rogue waves lies in their unpredictability, making them a

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major topic for research in oceanography and fluid dynamics [2].

Gerstner waves are often mentioned in rogue wave studies due to their special characteristics as an exact solution to the Euler equations for fluid motion. Gerstner waves are a special kind of trochoid, representing one of the few exact solutions available for deep-water waves. Due to their exactly solvable nature, Gerstner waves offer important insight into energy concentration and wave stability in a nonlinear context. Their role in rogue wave studies has emphasized many aspects of nonlinear effects, namely modulation instability, and how wave-current interactions may spontaneously trigger rogue waves.

The historical significance of Gerstner waves in fluid dynamics and wave theory dates to the early 19th century. Franz Josef von Gerstner was the first to present an analytic formulation for nonlinear waves in 1802—stationary solutions in the form of a trochoidal profile over deep water [3]. Though often regarded as more curious than directly applicable, this discovery was made almost a century before the famous Korteweg-de Vries (KdV) equation, laying a foundation for understanding nonlinear wave behavior. Unlike the KdV equation, which considers solitons in the regime of small nonlinearity and dispersion, Gerstner’s wave represents an exact solution to the full hydrodynamic equations without approximation for gravity waves on deep water.

Despite its mathematical elegance, Gerstner’s contribution to nonlinear wave theory has been historically underappreciated, often mentioned only in passing by authors focused on other approaches. The exact solution of Gerstner retains significant mathematical and physical value because it enables investigation of fluid dynamics beyond linear and weakly nonlinear theories. In the context of rogue waves—mysterious, nonlinear, and characterized by great randomness—Gerstner’s work resonates with contemporary challenges in predicting and understanding extreme wave events. Studying Gerstner waves helps derive analytical tools and concepts that form part of a larger framework indispensable for scientists involved in modeling and analyzing rogue waves, advancing our understanding of some of the ocean’s most powerful phenomena.

A dramatic and influential event in wave research occurred when John

Scott Russell observed a “wave of translation” on Edinburgh’s Union Canal in 1834 [4]. Russell saw a wave persist in moving forward after a canal boat had stopped, maintaining its form and velocity—a behavior not explained by the wave theories of his time. Russell pursued this wave on horseback, analyzing its characteristics and ultimately identifying what we now recognize as the soliton wave [5]. Russell’s pursuit and analysis of this solitary wave laid the groundwork for modern interpretations of rogue waves, emphasizing the importance of nonlinear wave phenomena. This realization not only challenged the validity of traditional linear wave models but also greatly influenced the development of mathematical frameworks for predicting and analyzing rogue wave formation, eventually leading to the Nonlinear Schrödinger Equation (NLSE).

Russell’s work on soliton waves provided a cornerstone for understanding nonlinear dynamics related to rogue wave emergence, leading to a reevaluation of ocean dynamics theories. These historical perspectives, in conjunction with modern computational methods, serve to deepen our understanding of rogue waves and help us avert their possible threats. Russell’s discovery remains an essential part of oceanographic studies and emphasizes the necessity for sophisticated models to forecast rogue waves.

It was not until the rogue wave recorded on January 1, 1995, at the Draupner oil platform in the North Sea that scientists confirmed their actual existence [6]. Rogue waves, once considered maritime myths and mere tales among mariners, had indeed created havoc and destruction throughout history. This single event marked a turning point in oceanography, as rogue waves could no longer be dismissed as nautical folklore.

Rogue waves are statistically rare, yet they have been observed and recorded in oceans throughout the world. Despite their rarity, they pose a significant danger to both the maritime and offshore industries, underscoring the need for comprehensive research and understanding. Earlier theories, prior to models such as the Nonlinear Schrödinger Equation (NLSE) and Gerstner’s equation, found it difficult to predict or even explain the formation of rogue waves. The linear superposition principle and the Rayleigh distribution of wave heights were

insufficient for explaining waves that exceeded twice the significant wave height.

This realization has made it necessary to incorporate rogue waves into maritime safety considerations, influencing the design of ships and offshore platforms. Current engineering standards require that such structures withstand forces beyond those predicted by conventional models, which did not account for the extreme impact of rogue waves [7]. This underlines the importance of integrating rogue wave data into design parameters to ensure higher safety margins for maritime structures.

The loss of the German cargo ship MS München in the Atlantic Ocean in December 1978 stands as a stark reminder of the destructive power of rogue waves. The catastrophic sinking prompted extensive investigation and has had an enormous impact on how maritime hazards are approached. This case has driven further research into rogue waves and influenced structural design standards to improve the survivability of ships encountering such conditions. In light of this tragedy, the need for continued research and adaptation of maritime practice is clear, as our understanding of rogue waves and the strategies to mitigate their risks must continue to evolve.

This paper is devoted to studying rogue waves mathematically, concentrating on the instability of nonlinear systems, with particular attention to the Nonlinear Schrödinger Equation (NLSE) and Gerstner's equation. We trace relevant aspects of these phenomena back to the Navier-Stokes equations and Euler's equation for fluids, aiming to contribute toward a practical understanding of rogue waves. By examining their historical context, statistical occurrence, and impact, we hope to advance both scientific understanding and progress in maritime safety and navigation protocol.

2. Key Equations

The main equations for fluid dynamics are the continuity equation and the momentum equation. These equations are the foundation of the equations to

come and are as follows;

$$\text{Continuity Equation: } \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0. \quad (1)$$

In this equation ρ represents the density, $\mathbf{v} = \frac{d\mathbf{x}}{dt}$ is the velocity and $\rho \mathbf{v}$ is the mass per area per time, referred to as the "flux." This equation describes conservation of mass: the rate of change of the mass is due to the gradient of the flux. When dealing with water waves, we assume incompressible flow. This in turn means that ρ is constant, therefore giving

$$\nabla \cdot \mathbf{v} = 0. \quad (2)$$

The momentum equation takes the form

$$\text{Momentum Equation: } \rho \frac{D\mathbf{v}}{Dt} = \mathbf{f}. \quad (3)$$

This equation is an adaptation of $\mathbf{F} = m\mathbf{a}$. In this equation \mathbf{f} represents the external force per volume which includes gravity, pressure, and viscosity.

3. Laws of Conservation

Conservation of Mass

Often included in the continuity equation, mass conservation simply states that the mass in a given system does not change with time. Mathematically, this amounts to stating the expression

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (4)$$

where $\mathbf{v} = \frac{d\mathbf{x}}{dt}$ is the velocity field. In simpler terms, this equation establishes that any change of mass in a control volume is entirely due to mass flux ($\rho \mathbf{v}$) at its boundaries. For incompressible flows, like those of water waves, the equation further reduces to reflect the constant mass density of the fluid, yielding the condition $\nabla \cdot \mathbf{v} = 0$. This simplification emphasizes the incompressibility of the fluid and points toward the preservation of volume within the flow.

Conservation of Momentum

Parallel to the conservation of mass, momentum conservation in a fluid is important for describing the forces and movement taking place in the fluid system. Similar to Newton's second law ($\mathbf{F} = m\mathbf{a}$), the momentum equation equates the unbalanced forces acting on a material volume with its acceleration. When applied to fluids, $\mathbf{F} = m\mathbf{a}$ takes the form

$$\rho \frac{D\mathbf{v}}{Dt} = \mathbf{f}, \quad (5)$$

where $\frac{D}{Dt}$ is the material derivative, representing the rate of change of velocity as one follows a given fluid parcel. The equation fully represents the influences of outside forces on the fluid acceleration, including gravitational forces, pressure gradients, and viscous effects. One such formulation is the Navier-Stokes equation, which accounts for fluid viscosity and provides a complete description of the dynamics of fluid flow under many different conditions. In other words, these laws of conservation underlie the analysis and solution of a variety of problems in fluid dynamics. From understanding natural phenomena of ocean waves to predicting flow around aerodynamic bodies, both require these fundamental principles.[8]

4. Linear Waves

The question of waves evidently represents one of the principal issues that touch on the discourse of fluid dynamics. While the nonlinear equations describe the general behavior of waves in fluids, linear approximations provide a useful solution to a large part of wave phenomena, in particular when dealing with small amplitude waves. This provides a basis for the approximate theory known as Airy wave theory. It was the British astronomer and mathematician Sir George Biddell Airy who made seminal contributions to the understanding of wave mechanics.

Central to the analysis of linear waves is the characterization of wave motion in terms of sinusoidal functions. Here, the equation that gives the profile of the

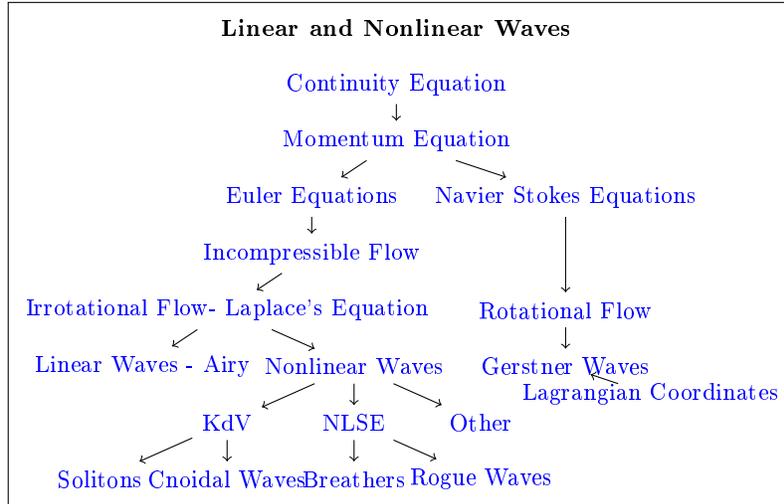


Figure 1: Diagram displaying the connection between different types of linear and nonlinear waves and the fundamental equations used in their derivation.

free surface elevation of the wave is

$$\eta(x, t) = A \cos(kx - \omega t + \phi). \quad (6)$$

In this expression, A denotes the amplitude, k is the wave number, and ω stands for the angular frequency. One more parameter that appears in the general solution of the differential equation is the phase. The wavenumber k is related to the wavelength λ through $k = \frac{2\pi}{\lambda}$; then, the angular frequency ω relating to the wave period T is $\omega = \frac{2\pi}{T}$, thus wrapping both time and space periodicity in the wave.

When analyzing waves in a fluid, one makes some assumptions about the behavior of the fluid; in particular, it is assumed to be incompressible and the flow to be irrotational. Such assumptions allow us to suppose that a velocity potential $\phi(x, z, t)$ can exist, having the property of being a solution of the Laplace equation,

$$\nabla^2 \phi = 0. \quad (7)$$

Boundary conditions are imposed at the ocean floor and the air-sea interface. At the bed ($z = -h$), the condition that the vertical velocity must vanish physically

translates into a no-flow condition. On the surface ($z = \eta(x, t)$), both kinematic and dynamic boundary conditions apply so that the fluid particles at the surface follow the wave form and that pressure across the air-sea interface is continuous.

The linearized boundary conditions are then imposed on the Laplace equation. As a consequence, the dispersion relation between the wave frequency ω and the wavenumber k is obtained. This relation is

$$\omega^2 = gk \tanh(kh), \tag{8}$$

which exhibits dependence on both the depth of the fluid and wavelength, showing a transition from deep to shallow water wave behavior. The linear dispersion relation allows predicting wave speeds and gives insight into phenomena such as wave refraction and dispersion of wave energy across different frequencies.

Far from merely academic curiosity, the theoretical framework for linear waves finds applications in the design of coastal and marine structures, wave propagation prediction for purposes of navigation and surfing, and assessment of wave energy potential as a renewable resource for power generation. In addition, this framework is relevant to understanding the phase velocity of waves, such as in the study of tsunamis, where their large wavelengths allow them to cross oceans at high velocities. Airy wave theory provides a foundational approach to studying linear waves in fluid dynamics, balancing mathematical rigor with physical applicability. Although greatly simplified, this linear approach serves as a starting point for the analysis of more complex wave phenomena, which arise under nonlinear dynamics and include waves that interact with varying topographies and currents.

5. Deep Water Waves

In regards to deep water waves, we assume the fluid domain to be a semi-infinite region expressed as

$$(x, y) : x \in \mathbb{R}, y < \eta(t, x) \tag{9}$$

which is bounded by η . If we let $(u(t, x, y), v(t, x, y))$ be the velocity field of the flow in the fluid domain and we also assume a constant density, or homogeneity, we arrive at the equation of mass conservation,

$$u_x + v_y = 0. \quad (10)$$

We can also assume no viscosity, or inviscid flow, which allows the motion of the waves to be given by Euler's equation. That is,

$$\begin{aligned} u_t + uu_x + vu_y &= -P_x, \\ v_t + uv_x + vv_y &= -P_y - g, \end{aligned} \quad (11)$$

where the pressure is given by $P(t, x, y)$ and g is the gravitational constant of acceleration. From here we introduce a dynamic boundary condition in order to decouple the motion of the air from the free particles at the surface. This can be shown as

$$P = P_0 \text{ on } y = \eta(t, x), \quad (12)$$

where P_0 represents the constant atmospheric pressure. Since the free surface will always be composed of the same particles, we can implement a kinematic boundary condition,

$$v = \eta_t + u\eta_x \text{ on } y = \eta(t, x). \quad (13)$$

The boundary condition for the bottom shows that as the depth of the water increases, the motion reduces. At great depths, there is almost no motion, which we can express as

$$(u, v) \longrightarrow (0, 0) \text{ as } y \longrightarrow -\infty, \text{ uniformly for } x \in \mathbb{R}, t \geq 0. \quad (14)$$

6. Nonlinear Schrödinger Equation

The Nonlinear Schrödinger Equation (NLSE), represented as

$$i \frac{\partial \Psi}{\partial t} + \beta \frac{\partial^2 \Psi}{\partial x^2} + \gamma |\Psi|^2 \Psi = 0, \quad (15)$$

where $\Psi(x, t)$ is the wave function representing the evolution of wave packets in nonlinear dispersive media. Here, β denotes dispersion, and γ stands for the intensity of the nonlinear interaction. This equation incorporates the interplay of dispersion while nonlinearity can focus and maintain the shape of the wave over large distances.

One would question how a formula that is featured mostly in quantum mechanics becomes relevant in the study of oceanic rogue waves. The connection arises because the NLSE governs the evolution of wave envelopes in dispersive, nonlinear media—a description that applies equally to quantum wave functions and to the modulation of ocean surface waves. The envelope of wave groups contains the individual waves in the group and gives insight into their stability, evolution, and potential to form rogue waves under certain conditions.

The NLSE has the capability to model nonlinear waves that travel in media such as fluids, plasmas, and optical devices. It unifies key concepts of wave dispersion and nonlinearity that are essential in the analysis of phenomena such as solitons. Exact solutions are important for the NLSE, as they give an accurate account of the complex interactions of waves and can help predict system behaviors under various conditions.

7. Split-Step Method

The Split-Step Fourier Method (SSFM) efficiently simulates the NLSE by separating the equation into its nonlinear and linear components, addressing them separately via the Fourier transform. Denoting the wave function as u for notational convenience, we first apply the nonlinear term, $u_t = i|u|^2u$, directly to the wave function in real space, modifying the wave by an exponential operator. This is followed by a Fourier transform to convert the wave function to frequency space. Here, the linear dispersion term, $u_t = iu_{xx}$, is managed by applying another exponential operator that handles wave dispersion effects. An inverse Fourier transform then returns the wave to real space. The process is repeated for each time step Δt , so that both the effects of nonlinearity and dispersion

are accurately modeled through time. We explored the NLSE across a spatial domain (L) of 100 units, discretized into 2^{15} grid points (N) for high resolution. Each spatial step measures $\frac{L}{N}$, and the simulation spans a total time of 10 units, with time steps Δt of 0.005 units for optimal resolution. The figure below shows the Peregrine solution—a known exact solution of the NLSE—deformed by added noise. By applying noise to the Peregrine solution, we can model rogue wave behavior more realistically, demonstrating the sensitivity and stability of the wave system: small alterations in the initial conditions can lead to significant changes in wave amplitude and behavior.

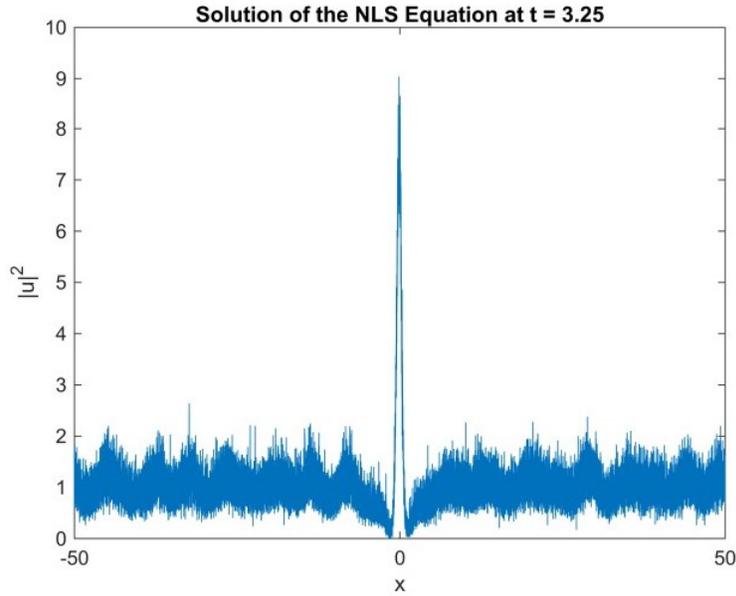


Figure 2: Peregrine solution of the NLSE with noise, computed via the Split-Step Fourier Method.

8. Peregrine Solution

The Peregrine solution,

$$\Psi(t, x) = \left(1 - \frac{4(1 + 2i\omega t)}{1 + 4\omega^2 t^2 + 4x^2} \right) e^{i\omega t}, \quad (16)$$

is a solution to the NLSE and is a widely recognized model for studying rogue waves. This solution is particularly notable for describing the sudden formation of extreme waves in seemingly calm seas. The Peregrine solution represents a localized disturbance that grows rapidly in amplitude, reaching a peak of approximately three times the background wave height, before decaying back to the uniform state—a behavior characteristic of “breather” solutions. This transient intensification and subsequent decay, all occurring within a relatively short period, mimics the sudden appearance and disappearance of rogue waves in nature. This model is invaluable for studying the dynamics and conditions under which rogue waves occur, offering insights into their prediction and the potential hazards they pose in marine environments.

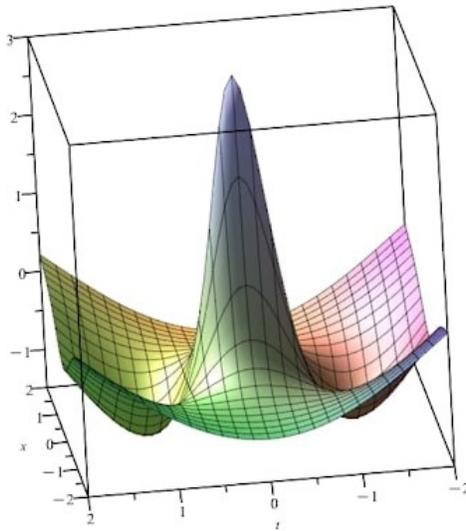


Figure 3: Three-dimensional spatiotemporal evolution of the Peregrine solution.

9. Gerstner’s Equation

Gerstner’s wave theory, also called the trochoidal theory of wave motion, deals with the behavior of water waves with respect to the surface. The surface

elevation can be expressed as

$$\zeta(x, t) = a \cos(kx - \omega t + \phi), \quad (17)$$

where ζ represents the surface elevation of the wave, a is the wave amplitude, k is the wave number, x is the horizontal position, ω is the angular frequency, and ϕ is the phase constant. While this expression resembles a linear wave profile, Gerstner's key contribution lies in the particle trajectory description given by the transformation equations below. Gerstner showed that the particles on the surface of an ocean move in circles (or ellipses), which has found important applications in the modeling of ocean waves. Gerstner waves are the only exact solutions for gravity waves on deep water in the case of incompressible fluid [7]. The shape of an ocean wave is often shown as a sine wave, but the actual wave profile is termed a trochoid. Although similar in shape to a sine wave for small amplitudes, a trochoid has a different form at large amplitudes, representing points traced by a circle rolling along a line. The shape features a narrowing or steepening of peaks that becomes more pronounced with increasing wave amplitude [9].

Gerstner waves are two-dimensional waves that adopt a Lagrangian viewpoint and require two parameters, $a \in \mathbb{R}$ and $b \leq b_0$ for a fixed $b_0 \leq 0$. Eulerian viewpoints tend to be simpler and therefore easier to work with; however, Abrashkin and Pelinovsky have published work showing the significance of Lagrangian coordinates in both Gerstner waves and the NLSE [5].

The lower half of the plane represents a motionless body of water. If we choose a specific particle by fixing a and b , the Gerstner wave is given by Gerstner's transformation,

$$\begin{aligned} x &= a + \frac{e^{mb}}{m} \sin\left(m\left(a + \sqrt{\frac{g}{m}} t\right)\right) \\ y &= b - \frac{e^{mb}}{m} \cos\left(m\left(a + \sqrt{\frac{g}{m}} t\right)\right) \end{aligned} \quad (18)$$

where $m > 0$ is fixed. This transformation traces the path of a single particle, which is circular, anticlockwise, centered at (a, b) , with a radius of $\frac{e^{mb}}{m}$ and an

angular velocity of $\sqrt{\frac{g}{m}}$. We can apply this for as many particles as desired by changing the values of a and b and iterating over a given interval. We can obtain the profile of a surface wave as time evolves by setting $b = b_0$, which gives

$$\begin{aligned} x &= a + \frac{e^{mb_0}}{m} \sin\left(m\left(a + \sqrt{\frac{g}{m}} t\right)\right) \\ y &= b_0 - \frac{e^{mb_0}}{m} \cos\left(m\left(a + \sqrt{\frac{g}{m}} t\right)\right) \end{aligned} \quad (19)$$

which describes a smooth trochoidal curve for $b \leq b_0$. If the still water surface is at $\{(a, 0) : a \in \mathbb{R}\}$, it would show the profile of a cycloid, a continuous curve with upward cusps.

The initial momentum equations are,

$$\begin{aligned} X_{tt}X_a + (Y_{tt} + g)Y_a &= -\frac{1}{\rho}P_a, \\ X_{tt}X_b + (Y_{tt} + g)Y_b &= -\frac{1}{\rho}P_b. \end{aligned} \quad (20)$$

These determine the acceleration of the fluid particles in both horizontal and vertical directions, considering the effects of gravitational acceleration (g) and the gradient of fluid pressure ($\frac{1}{\rho}P$) on the motion. Here X_{tt} and Y_{tt} denote the second derivatives of the particle positions X and Y with respect to time, indicating acceleration, while P_a and P_b represent the pressure gradient components along the directions parameterized by a and b .

To further understand the context in which these equations apply, we take the curl of the momentum equations to obtain information pertaining to the vorticity dynamics of the fluid:

$$0 = \frac{\partial}{\partial b}(X_{tt}X_a + (Y_{tt} + g)Y_a) - \frac{\partial}{\partial a}(X_{tt}X_b + (Y_{tt} + g)Y_b). \quad (21)$$

This simplifies by the chain rule and commutation of the partial derivatives to an expression involving the mixed second derivatives of X and Y with respect to t , a , and b . This term dictates that, due to the conservation of momentum, there is no net rotational acceleration in the fluid, a feature arising from the conservation of vorticity.

$$0 = X_{ttb}X_a + Y_{ttb}Y_a - (X_{tta}X_b + Y_{tta}Y_b), \quad (22)$$

which can be interpreted as the rate of change of a quantity related to the vorticity of the fluid. Further manipulation, integration with respect to time, and introduction of a vorticity function $\Omega(a, b)$, defined by

$$X_{tb}X_a + Y_{tb}Y_a - X_{ta}X_b - Y_{ta}Y_b = \Omega(a, b), \quad (23)$$

indicates that the vorticity of the fluid Ω is constant over time for any particle path initialized by a and b . Gerstner's wave solution is encompassed by the equations

$$\begin{aligned} X &= a + \frac{1}{k}e^{kb} \sin(k(a + ct)), \\ Y &= b - \frac{1}{k}e^{kb} \cos(k(a + ct)). \end{aligned} \quad (24)$$

These describe the paths of fluid particles in a Gerstner wave. This solution exhibits the circular motion of fluid particles under the influence of the wave, where k is the wave number and c is the phase speed of the wave. The exponential term e^{kb} accommodates the decay in motion with depth below the free surface, while the sinusoidal terms identify the circular particle orbits characteristic of Gerstner wave theory.

This derivation, leading from the momentum equations to Gerstner's solution, underlines the nontrivial nature of the fluid dynamics of nonlinear wave motion.

The Gerstner waves trace distinct trochoidal wave profiles. Each peak and trough follows the form of a trochoid—the extended cycloid curve. Gerstner wave solutions mark particles by parameters a and b and follow the paths given by the coordinates (X, Y) in time, tracing circles at each depth with the center at (a, b) . The orbital radii decrease with depth. Gerstner waves evolve from basic sinusoidal descriptions to capture a much more accurate representation of wave motion in fluid dynamics, including the rotational component of water particles in each wave. The resulting profiles exhibit nonlinear, non-dispersive behavior. The precise modeling seen in the figures provides insight into the

behavior of gravitational waves, and the correlations of wave parameters that are relevant to the study of rogue waves and nonlinear dynamics.

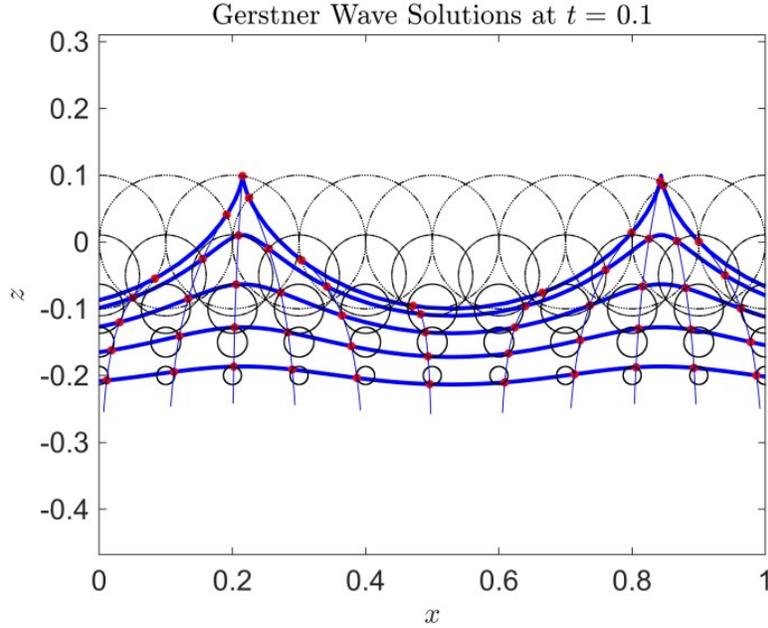


Figure 4: Gerstner wave solutions plotted in two dimensions, showing trochoidal profiles with circular particle orbits at varying depths.

10. Closing Remarks

Our study advances the understanding of rogue wave dynamics through complementary analysis using both Gerstner wave theory and NLSE solutions. These approaches supply valuable knowledge about how rogue waves could form and behave, underlining the unpredictability and danger that they represent. The research underscores an essential need for better predictive models to improve safety for mariners and critical infrastructure.

Further development of these models will lead to more accurate prediction of rogue waves. This will involve incorporating actual oceanographic data within our models to validate and refine our theoretical approaches by aligning

theoretical insight with empirical data. Additionally, we aim to explore machine learning approaches to rogue wave prediction. Machine learning applied to historical data and real-time ocean conditions could identify patterns that precede rogue wave occurrence. Such interdisciplinary approaches would further improve our understanding of rogue waves and enable more effective strategies to mitigate risks in maritime operations. Ultimately, our aim is to leverage scientific innovation to protect lives and assets from the dangers of rogue waves.

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